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**Hp-ADAPTIVE FINITE ELEMENT METHODS FOR TIME  
DEPENDENT PROBLEMS WITH APPLICATIONS TO  
STRESS WAVES IN SOLIDS**

**FINAL PROGRESS REPORT**

**Professor J. Tinsley Oden  
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**August, 1996**

**U.S. ARMY RESEARCH OFFICE**

**DAAL03-92-G-0253**

**THE UNIVERSITY OF TEXAS AT AUSTIN**

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## FINAL PROGRESS REPORT DAAL03-92-G-0253

### *Statement of Problem Studied*

The general goals of this project included the development of new, high-order, adaptive methods for the computer simulation of stress-wave propagation phenomena in solid mechanics, particularly elastodynamics.

The motivation for using adaptive methodologies is to control and optimize the computational process, to use a posteriori error estimates to optimize meshes and spectral orders of approximation. Such orchestrated meshing can produce exponential rates of convergence, thereby allowing complex simulations to be done using orders-of-magnitude fewer unknowns than standard methods.

### *Summary of Results*

This project has led to a number of completely new and powerful methods for the computer simulation of complex problems, including, in particular, problems of wave propagation. These include the following:

- High-Order Multistage-Taylor Galerkin Methods. These represent one of the only new unconditionally stable high-order time integration schemes developed in decades. They are designed to overcome a well-documented deficiency of splitting methods: the loss of time accuracy due to splitting of boundary conditions.

- A Posteriori Error Estimation Methods. New techniques for the estimation of error in numerical approximations of wave problems in two space dimensions have been developed; theorems have been established to guarantee that rigorous error bounds are possible.

- Adaptive methods. Local error estimates provide data for adaptively changing mesh sizes and spectral orders to optimize hp meshes and accelerate convergence.

- Parallel Discontinuous Methods. A surprisingly parallelizable scheme based on Discontinuous Galerkin methods has been developed and tested on model problems.

- Clouds: A New family of Meshless Methods. Error estimates and preliminary results on the mathematical foundations of a new type of meshless technique for solving partial differential equations have been established.

### *List of Publications*

Safjan, A. and Oden, J.T., "High Order Taylor-Galerkin and Adaptive hp Methods for Second-Order Hyperbolic Systems: Application to

- Elastodynamics," *Computer Methods in Applied Mechanics and Engineering*, v. 103, nos. 102, pp. 187-230.
- Ainsworth, M. and Oden, J.T., "A Procedure for A Posteriori Error Estimation for hp-Finite Element Methods," *Computer Methods in Applied Mechanics and Engineering*, Vol. 101, 1992, pp. 73-96.
- Ainsworth, M., Oden, J.T., and Wu, W., "A Posteriori Error Estimation for hp-Approximations in Elastostatics," *J. Appl. Num. Math.*, 14, 1994, pp. 23-54.
- Oden, J.T., "Optimal hp-Finite Element Methods," TICOM Report 92-09
- Oden, J.T., "Optimal hp-Finite Element Methods," *Computer Methods in Applied Mechanics and Engineering*, vol. 112, 1994, pp. 309-351.
- Safjan, A. and Oden, J.T., "High-Order Taylor-Galerkin and Adaptive hp-Methods for Hyperbolic Systems," *J. Computational Physics*, vo. 120, 1995, pp. 206-230.
- Oden, J.T., Safjan, A., Geng, P. and Demkowicz, L., "High-Order, Multi-Level Adaptive Time-Domain Methods for Structural Acoustics Simulations," *Large-Scale Structures in Acoustics and Electromagnetics*, National Academy of Sciences, September 1994, TICAM Report 94-14.
- Duarte, C. Armando and Oden, J.T., "H-p Clouds -- an h-p Meshless Methods," *Numerical Methods for Partial Differential Equations*, Vol. (1996), (in press).
- Duarte, C. Armando and Oden, J.T., "A New Meshless Method to Solve Boundary-Value Problems," Proceedings of the XVI CILAMCE-Iberian Latin American Conference on Computational Methods for Engineering, Curitiba, Brazil, November, Machado, R.D. (ed)., 1995, pp. 90-95.
- Duarte, C. Armando and Oden, J.T., "Hp Clouds -- A Meshless Method to Solve Boundary-Value Problems," TICAM Report 95-05, May 1995.
- Duarte, C. Armando and Oden, J.T., "An h-p Adaptive Method Using Clouds," TICAM Report 96-07, February 1996.

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